4-4 day 3 Optimization: Economics

Learning Objectives:

I can use derivatives to identify to optimize quantities in real world situations.

Economics

x = The number of items produced and sold

r(x) = The revenue generated from selling x items

c(x) = The cost ~~from~~ of making x items

p(x) = The profit from making and selling x items

$$p(x) = r(x) - c(x)$$

Economics

$$\frac{\Delta r}{\Delta x}$$

$$\frac{dr}{dx} =$$ *The rate of change of the revenue*

The Marginal Revenue (MR)

$$\frac{dc}{dx} =$$ *The rate of change of the cost*

The Marginal Cost (MC)

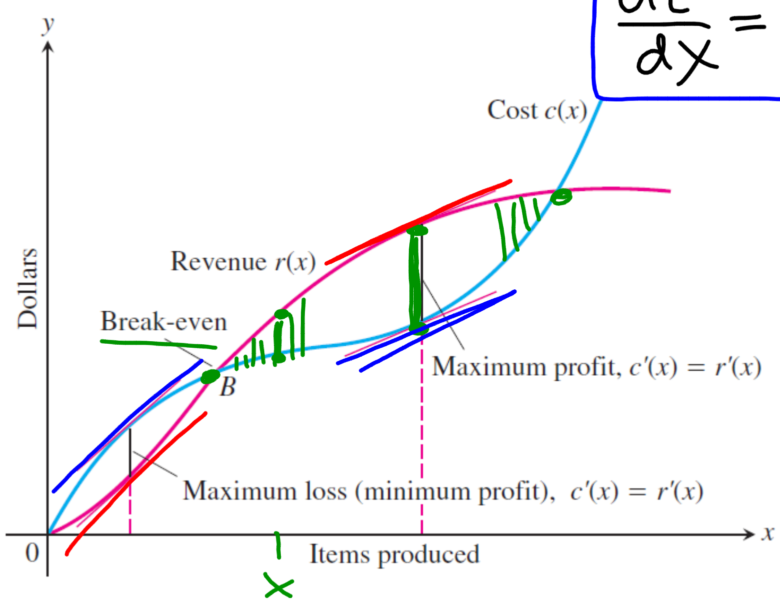
$$\frac{dp}{dx} =$$ *The rate of change of the profit*

Oct 30-11:13 AM

Maximum Profit

The maximum profit occurs at a production level at which the marginal cost equals the marginal revenue. $MC = MR$

$$\boxed{\frac{dc}{dx} = \frac{dR}{dx}}$$



$$p(x) = r(x) - c(x)$$

$$\frac{dp}{dx} = \frac{dr}{dx} - \frac{dc}{dx}$$

$$0 = \frac{dr}{dx} - \frac{dc}{dx}$$

$$\boxed{\frac{dc}{dx} = \frac{dr}{dx}}$$

Ex1. You start a garage band with some of your friends. You record an album using the Garage Band software and sell CD's for $10 each in the lunchroom. The cost of producing x CD's is $c(x) = .009x^2 + 3.083x + 152.201$

$r(x) = 10x$

State ordinances allows garage bands to produce at most 500 CD's independently before requiring bands to have a record label. You have not signed a record deal.

a.) What production level will maximize the profits? What is the profit at this production level?

Method #1 $P'(x)=0$

1. \$10 per CD cost to make: $c(x)=.009x^2+3.083x+152.201$ $[0,500]$

$r(x)=10x$ $c(x)=.009x^2+3.083x+152.201$ ep.: 0, 500

$P(x)=10x-(.009x^2+3.083x+152.201)$ der=0: 384

$P(x)=-.009x^2+6.917x-152.201$

$P'(x)=-.018x+6.917$ $P(0)=-152.201$

$0=-.018x+6.917$ $P(500)=$

$x=384$ cd's

$P''=-.018$ max

Method #2 $c'(x)=r'(x)$

- ie. sell CD's for \$10, cost to make is

$c(x)=.009x^2+3.083x+152.201$ $r(x)=10x$

must stop at 500 CDs $[0,500]$

$10=.018x+3.083$ $6.917=.018x$ $x=384.278$

- Avg Cost $=\frac{c(x)}{x}$ (384, 1176.82)

Candidates

endpts: $x=0,500$

der: $x=384$

x	p(x) \$
0	-152.20
384	\$1176.81
500	\$1056.30

Max

b.) What production level will minimize your average cost? What is your average cost at this production level?

$$\text{Avg Cost} = \frac{C(x)}{x}$$

$$\text{Avg Cost} = \frac{.009x^2 + 3.083x + 152.201}{x}$$

$$\text{Avg Cost} = .009x + 3.083 + \frac{152.201}{x}$$

$$AC = .009x + 3.083 + 152.201x^{-1}$$

$$AC' = .009 - 152.201x^{-2}$$

$$\left(0 = .009 - \frac{152.201}{x^2}\right)x^2$$

$$0 = .009x^2 - 152.201$$

$$\frac{152.201}{.009} = \frac{\cancel{.009}x^2}{\cancel{.009}}$$

$$\sqrt{16,911.22} = \sqrt{x^2}$$

$$x \approx 130 \text{ CD's}$$

$$AC'' = 304.402x^{-3}$$

$$AC'' = \frac{304.402}{x^3}$$

$+$

ccu

c.) Find the production level at which the average cost equals the marginal cost?

Avg Cost $= .009x + 3.0834\ \frac{152.201}{x}$

$C'(x) = .018x + 3.083$

$.009x + \cancel{3.083} + \frac{152.201}{x} = .018x + \cancel{3.083}$

$\left[.009x + \frac{152.201}{x} = .018x \right] x$

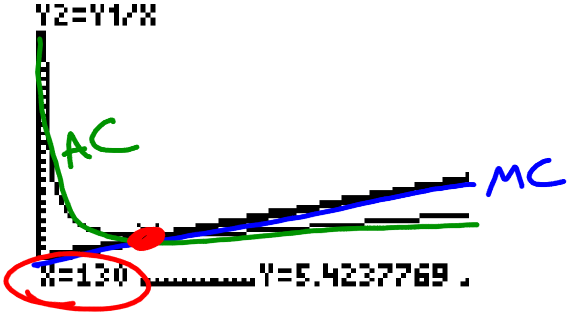
$\cancel{.009x^2} + 152.201 = .018x^2$
$\cancel{-.009x^2} \qquad\qquad -.009x^2$

$152.201 = .009x^2$

⋮ same as before hmmh?

$x = 130$

d.) Something special happens at this production level. Make a hypotheses as to what that is.



When Avg Cost is a Min,

$AC = MC$

$$\frac{C(x)}{x} = C'(x)$$

This is the only point when this happens and it always happens at this pt.

Oct 31-2:16 PM

Homework

pg 226 # 23-26, 51, 52, 54, 62